Physics 22 Practice Midterm - 50 minutes
3 pages

Harry Nelson

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Write your answers in a blue book. Calculators and one page of notes allowed. No textbooks allowed. Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Make it clear what you think is known, and what is unknown and to be solved for. Except for extremely simple problems, derive symbolic answers, and then plug in numbers (if necessary) after a symbolic answer is available. **Put a box around your final answer... otherwise we may be confused about which answer you really mean, and you could lose credit.**

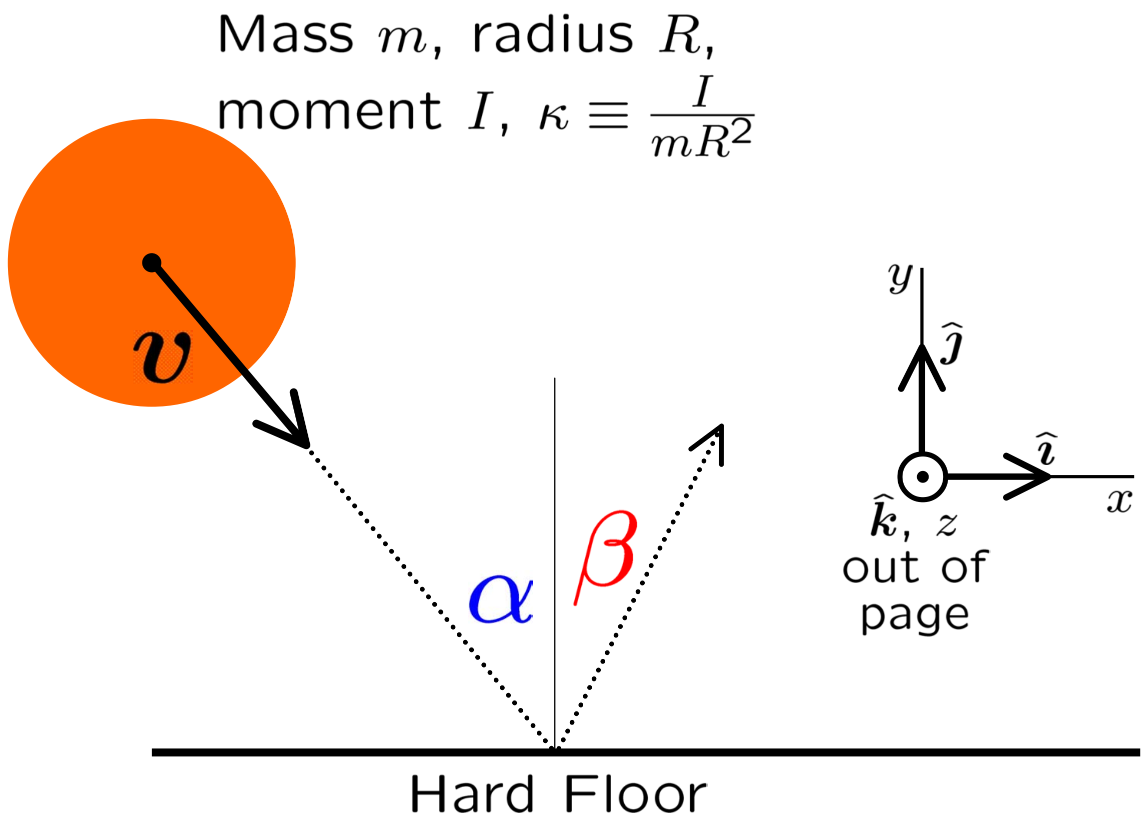

Figure 1: For use in problem 1.

1. A ball with initial velocity $\boldsymbol{v} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}}$ is about to hit the floor, as shown in Fig. 1. The ball has mass m, radius R, and moment of inertia I. The reduced moment of inertia is defined by $\kappa \equiv I/(mR^2)$. Just

before the bounce the velocity makes an angle α with the vertical, as shown in Fig. 1. Neglect gravity, because the time between what is shown in Fig. 1 and the bounce is so small that gravity has no chance to make a difference. The bounce of the ball off the floor is elastic, meaning that the magnitude of the vertical component of the velocity after the bounce is the same as that before the bounce, but opposite in direction. The ball is initially not spinning at all. There is a lot of friction between the ball and the floor, so spin influences the outcome of the bounce. The ball never slips during the bounce so friction does no work on the ball and the ball's final total energy equals its initial total energy. Denote the final velocity of the ball by $\boldsymbol{s} = s_x\hat{\boldsymbol{\imath}} + s_y\hat{\boldsymbol{\jmath}}$, and the final angular velocity of the ball by $\boldsymbol{\omega}$. Use the x, y, z coordinate system shown in Fig. 1.

(a) The impulse on the ball in the x direction from friction with the floor during the bounce is $J_x = \int F_x dt$.

  i. Is $J_x > 0$, $J_x = 0$, or $J_x < 0$, where the positive x direction is shown in the figure?
  ii. State the equation that describes the change in horizontal linear momentum due to J_x.
  iii. State the equation that describes the change in angular momentum due to J_x; be careful to state the point about which you evaluate angular momenta.
  iv. State the equation that describes conservation of energy before and after the bounce.

(b) Describe the direction of the angular velocity of the ball after the bounce as a vector, using the unit vectors shown.

(c) What is the final angular velocity vector, $\boldsymbol{\omega}$, in terms of the quantities that specify this problem?

(d) What is $\tan\beta$, in terms of the quantities that specify this problem?

2. A person rides a unicycle in a circle of radius R, as shown in Fig. 2. The total mass of the unicycle and the person is μ, and the center of mass of the whole system is a distance h above the bottom of the tire. They go at speed $|\boldsymbol{v}|$. The radius of the wheel is b, its mass is m, and its moment of inertia for spinning about its axis is $I = mb^2$. The acceleration of gravity is g. Find the angle ϕ that the unicycle is tipped at, as shown on Fig. 2, in terms of the other quantities that specify the problem.

3. When a certain spaceship of mass m is extremely far from a planet of mass $M_p \gg m$ and radius R_p, the ships aims itself to miss the edge of the planet by a distance h, and has speed v. On the surface of the planet, the acceleration of gravity is $g_p = GM_p/R_p^2$. As the ship nears the planet, the ship slows itself by bouncing a tractor beam aimed at the center of the planet (this is very efficient, since the ship slows down once by firing the tractor beam at the planet, and then gets a bonus on the rebound when the tractor beam bounced off the planet hits the ship). Eventually the ship ends up in a circular orbit; what is the radius r_0 of this circular orbit in terms of the other quantities that specify the problem? Eliminate GM_p in favor of g_p and R_p.

4. The following problems all involve the manipulation of complex numbers, where cartesian form is $z = x + iy$ and polar form is $z = re^{i\theta}$.

(a) Put in cartesian form: $z = 1/(-1+i)$

(b) Put in polar form: $z = -3 - 3\sqrt{3}i$

(c) Put in polar form: $z = \sqrt[3]{-i}$

(d) Put in the form $A\sin\omega t + B\cos\omega t$: $\mathrm{Re}(\sqrt{2}ie^{i\omega t - i\pi/4})$

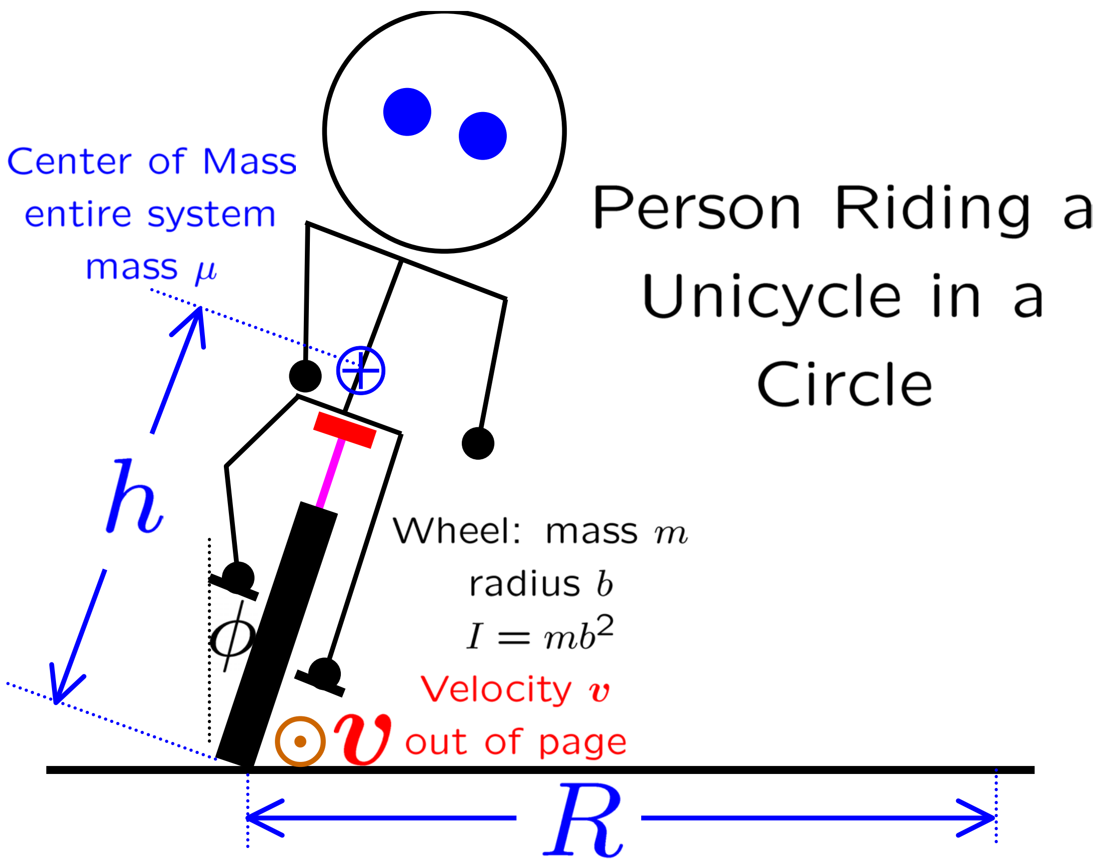

Figure 2: For use in problem 2.