Physics 25
Final Exam: June 13, 2007

The exam will run for 3 hours. The exam is open book - you may consult any book or set of notes that you wish. Use of a calculator is also permitted. The exam consists of **six** problems - you should answer all of them to obtain full marks. Each problem counts for an equal number of points. To obtain full credit, remember to show your work and draw a box around your answer to each question.

The exam counts for 50% of the total marks in the course.

Some Possibly Useful Information

Speed of light $c = 3.00 \times 10^8$ ms^{-1}.

Magnitude of charge of electron $e = 1.60 \times 10^{-19}$ C.

Mass of proton $m_p = 1.67 \times 10^{-27}$ kg.

Mass of electron $m_e = 9.11 \times 10^{-31}$ kg=0.511 MeV/c^2.

Permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}.

$k = 1/(4\pi\epsilon_0) = 8.99 \times 10^9$ Nm^{2}C^{-2}.

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$Ns^{2}C^{-2}.

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}.

Acceleration due to gravity near the earth's surface $g = 9.81$ ms^{-2}.

Boltzmann constant $k = 1.38 \times 10^{-23}$ JK^{-1}.

Planck's constant $h = 6.63 \times 10^{-34}$ Js.

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ Wm^{-2}K^{-4}.

Electron g-factor $g_e = 2.00$.

Bohr magneton $\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}$Am^2.

1.(a) An observer sits at rest in frame S. Two sources of light are also at rest in this frame, one at 1.50 meters directly in front of the observer, and the other 1.50 meters directly behind. Both sources emit light flashes simultaneously at $t = 0$ in this frame. When does the front flash reach the observer? When does the back flash reach the observer?

(b) An observer in frame S′ moves at constant velocity 2.50×10^8 m/s with respect to frame S along the straight line joining the light sources and the previous observer, toward the front light source. Both observers synchronize their watches as they pass each other at time $t = t' = 0$. In the frame of S′, when does the front flash reach the observer S′? In the frame of S′, when does the back flash reach the observer S′?

2. A gamma-ray photon of initial energy 2.50 MeV Compton scatters through 90° off an electron initially at rest.

(a) What is the final energy of the photon?

(b) What is the final kinetic energy of the electron?

(c) What is the final speed of the electron?

(d) What is the final momentum (in magnitude and direction) of the electron?

3.(a) What are the possible values of the four quantum numbers of a p electron in a hydrogen atom? What are the possible values of the total orbital angular momentum of such an electron? What are the possible values of the projection of this angular momentum onto some axis? What are the possible values of the projection of the total (including spin) angular momentum onto some axis? (Make sure that your answers are in units of angular momentum.)

(b) Two electrons are moving from left to right and encounter a potential barrier of thickness 6.0×10^{-10} m and "height" 7.6 eV. One of the electrons has an initial kinetic energy of 18 eV, and the other has an initial kinetic energy of 5.2 eV. Sketch two qualitative graphs showing the wave function of each electron as a function of distance, being careful to indicate the regions to the left, inside, and to the right of the barrier. What is the probability that the 5.2 eV electron will tunnel through the barrier?

4. A commonly used analytic approximation for the interaction potential energy between two atoms in a diatomic molecule is the "Lennard-Jones 6-12 potential":

$$U(r) = \epsilon \left[\left(\frac{a_0}{r} \right)^{12} - 2 \left(\frac{a_0}{r} \right)^{6} \right],$$

where ϵ is a constant with dimensions of energy, a_0 is a constant with units of length, and r is the distance between the nuclei of the atoms.

(a) Derive an expression for the equilibrium separation of the two atoms.

(b) Given that the reduced mass of the atoms in this molecule is m_r, what would be the wavelength of the emitted photon if the molecule made a transition from the first excited vibrational level to the ground state, assuming no rotation in either state?

(c) What would be the wavelength of the photon produced in the rotational $J = 2 \rightarrow J = 1$ transition of the molecule within the lowest vibrational state?

5. (a) The heat capacities per molecule at constant volume for gaseous helium, nitrogen, and water vapor are given in the table. Describe briefly how the equipartion theorem explains these results.

He	$3k/2$
N_2	$5k/2$
H_2O	$3k$

(k is Boltzmann's constant.)

(b) A sample of pure crystalline silicon (a semiconductor with atomic number 14) is doped with a small amount of aluminum impurity atoms (atomic number 13). Sketch a qualitative diagram of the resulting electron energy band structure. Would you expect this to be a p-type, n-type, or some other type of semiconductor?

(c) Recall that in a free electron gas, the number of states in a volume V with momentum less than p is given by

$$N(< p) = \frac{V}{3\pi^2 \hbar^3} p^3 \left(\frac{g_s}{2} \right)$$

where $g_s = 2$ is the number of spin states for each electron. Use this formula to derive an expression for the electron Fermi energy E_{F} at zero temperature in terms of the number density n of electrons, assuming that the electrons are ultrarelativistic (energies $E >> m_e c^2$). (Hint: what is the relationship between energy and momentum for ultrarelativistic particles?)

6.(a) Fill in the information (indicated by double question marks) which is missing from the following nuclear reactions:

$$^{2}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H} \rightarrow ^{??}_{??}\mathrm{He} + \gamma$$

$$^{7}_{3}\mathrm{Li} + ?? \rightarrow 2^{4}_{2}\mathrm{He}$$

$$^{15}_{8}\mathrm{O} \rightarrow ^{15}_{7}\mathrm{N} + ?? + \nu_e$$

$$^{239}_{94}\mathrm{Pu} + ^{4}_{2}\mathrm{He} \rightarrow ^{241}_{96}\mathrm{Cm} + 2??$$

(b) NASA commonly uses "radioisotope thermoelectric generators" (RTGs) as a power source for space probes going to the outer solar system, where the sun is too faint for solar power. The RTGs aboard the Voyager spacecraft were based on the α-decay of plutonium-238,

$$^{238}_{94}\mathrm{Pu} \rightarrow ^{234}_{92}\mathrm{U} + ^{4}_{2}\mathrm{He},$$

with a half-life of 87.7 years. Given that the masses in atomic mass units of the three isotopes in this reaction are 238.0496 (Pu), 234.0410 (U), and 4.0026 (He), where one atomic mass unit is 1.66×10^{-27} kg, how much energy is released in each radioactive decay? Assuming that all that energy is converted into useful power by the generator at 100 percent efficiency, what is the mass of plutonium needed to supply the Voyager spacecraft with its power requirements (420 W)? Voyager 1 was launched in 1977 (30 years ago). How much power is its RTG producing now, if it was producing 420 W when it was launched?